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Analysis of Turbine Blades Using a Rapid Three-Dimensional Photoelastic Method

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A new fast-curing photoelastic material formulation is presented that, when used with recommended curing and loading cycles, reduces the time needed to obtain three-dimensional stress results to the order of a week or less. The new formulation allows models to be cast to shape (replicated from a prototype) rather than machined. This is a major factor in the reduction of time. The stress analysis of the dovetail joint of a turbine blade and fan disk is reported as an example of the new method.

Introduction

SINCE its inception in the last century, photoelasticity has been used as an alternate to theoretical methods to determine the stresses in bodies under load. Like other experimental methods of measurement, photoelasticity has been found most useful for problems in which the geometry of loading, or the geometry per se, was too complicated to permit solution of the boundary value problem of the governing partial differential equations. For those problems which do lend themselves to mathematical expression, theoretical solutions usually have a generality far surpassing the corresponding photoelastic solution. So both theory and experiment serve different classes of problems, with a certain amount of overlapping.

The introduction of three-dimensional photoelasticity broadened the scope of experimental stress analysis. The invention of the computer greatly widened the scope of theoretical stress analysis methods. It was felt by some that the introduction of computer-based methods, and the finite-element method in particular, rendered two-dimensional photoelasticity obsolete and as well three-dimensional photoelastic analysis of axisymmetric problems. The only exception was the use of photoelastic methods to check the finite-element methods.

Note should be taken of the fact that numerical stress analysis methods, while usually considered theoretical, have taken on several of the traits of experimental methods. As with experimental methods they often provide only a single (nonparametric) solution, and as with some measurement methods they provide the average stress over some finite area.

Three-dimensional stress analysis of nonaxisymmetric problems is difficult, experimentally or theoretically. Thus, for the historic reasons cited above and because of developments in three-dimensional photoelasticity, it has become a major area of interest in experimental stress analysis.

A singular difficulty in three-dimensional photoelasticity is the manufacture of the model. In most studies in order to avoid the rind effect, it has been felt necessary to machine the complete model from a single piece of photoelastic material or to cement several machined pieces together. Rind will be defined here as those spurious photoelastic fringes (isochromatics) generated on the cast surface during, and by, the curing process which cannot be eliminated by annealing. This study proposes the direct casting of the model, especially in those cases such as turbine blades where machining is difficult. A new material formulation is proposed which essentially eliminates the rind effect.

The suggested formulation, casting techniques, and accompanying modeling and loading techniques have all been designed to minimize the total analysis time, so as to complete the analysis in less than a week and thus optimize its value in industrial applications. The methods are illustrated with the analysis of a turbine blade joint under various loads. Besides illustrating the methods, the analysis was used in redesign of the joint and may be of interest to designers.

Experimental Procedures

Mold Making

For as-cast models, mold making becomes significant. One investigator[1] suggested steel molds. Although successful, the method requires machining often as difficult and time consuming as machining the model directly. Where a prototype of the body is available, silicone rubber molds have been used.[2-4]

Two mold-making procedures are suggested here. First, it is suggested making a seamless silicone rubber outer jacket, large enough to accommodate the body to be analyzed. This can be done by placing a core in a container and pouring silicone rubber around the core. When cured the core is removed from the container. Wooden blocks were used for cores and tin cans and Plexiglas boxes for containers. The jacket walls should be thick enough to resist bulging in subsequent operations (about 20 mm).

Once the jacket is made and cured, the prototype is placed in it and silicone rubber poured around it. After curing, the jacket is removed and the encapsulated prototype taken out. The mold material can deform greatly to facilitate removal of the prototype; however, where necessary, the mold can be cut (or even torn). The mold is fitted back together and replaced in the jacket, ready for casting the photoelastic model. Figure 1 shows such a mold. A number of variations of this procedure have been used, such as casting the inner mold in

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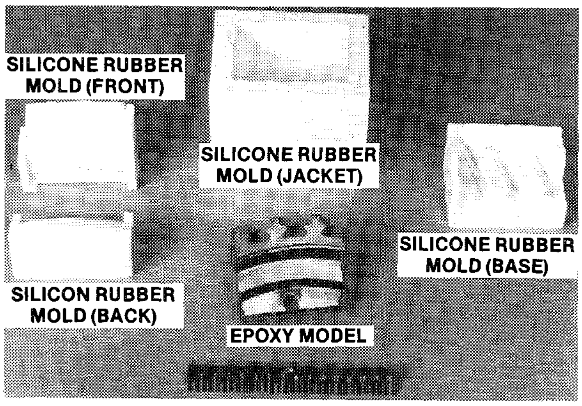

Fig. 1 Epoxy model disk surrounded by silicone rubber mold parts.

parts to avoid cutting and/or making the inner mold prior to the jacket.

The second procedure produces a rigid mold with an inner silicone rubber lining. A viscous paste is formed with two parts by weight silicone rubber and one part plaster of paris. A few drops of water (5 drops per 100 g of plaster) are added to the paste to accelerate the curing. The paste is brushed directly on the prototype to a thickness of about 2 mm. Permeable cloth, such as cheese cloth, is laid over the paste. The silicone rubber paste penetrates the cloth, forming a mechanical bond between the cloth and silicone rubber. After the mixture of silicone rubber, plaster of paris, and water is cured (this takes 1-2 h), plaster of paris is applied over the cloth to reinforce the mold. As an alternate to the plaster of paris layer, a syntatic foam of epoxy with hollow glass beads (manufactured by Furane Corporation, Los Angeles, Calif.) can be used. The mold is allowed to cure and then cut as necessary to remove the prototype. After removing the prototype, the parts of the mold are reassembled and any seams sealed with a bead of fast-curing silicone rubber. This second procedure is somewhat more complicated but it saves time. A moderately complicated mold can be prepared in 24 h. Reduction of the cost of mold materials can also be substantial.

In either procedure, if the complete prototype is replicated, pouring holes and vent holes must be provided. This can be done in casting the mold or by machining holes afterward.

The recommended silicone rubber is Dow Corning 3110 RTV with curing agent 1. The fast-curing silicone is made with curing agent 4. General Electric's RTV-11 was also used in this study but gave more edge effect in the photoelastic models. Various parting agents were used in making the molds and models. Teflon spray was applied to the wooden cores. Nothing was used on plexiglas and metal containers. A 10% petroleum jelly/90% methylene chloride solution was applied to the inside of the silicone jacket. The metal prototypes were sometimes sprayed with Teflon and other times not coated at all. The silicone molds were not coated for casting the photoelastic models.

The final curing of the molds is for 5-6 h at a temperature of 5-10°C above the curing temperature of the photoelastic materials. The final curing can be incorporated into the photoelastic casting cycle by heating the molds prior to mixing the photoelastic material, and then cooling to the temperature at which the photoelastic material is poured, as discussed below.

The Photoelastic Material

The formula that is recommended for the photoelastic material is: Epon 828 epoxy resin + phthalic anhydride (50% of the resin weight) + Hardener H (1% of the total resin and phthalic anhydride weight). Epon 828 is a Shell Chemicals product and can be obtained from a number of suppliers. Phthalic anhydride can be obtained as such from chemical suppliers or by trade names such as Hysol 4368 (Houghton Labs., Olean, N.Y.) or CA-2 (Stress-Strain Laboratories, Dallas, Texas). Hardener H is available as CA-1 (S-S Laboratories). It is recommended that Hardener H be aged by heating for 24 h at 93°C (200°F). The loss due to aging is about 11% of the original weight.

This formula is similar to formulas previously used, except for the addition of Hardener H. This addition is essential for rapid curing and elimination of unwanted rind on the cast surfaces.

Hardener H is added to the epoxy resin at room temperature and stirred several minutes until mixed. If Hardener H is crystalline it can be liquified by heating to 65°C. Hardener H is slightly toxic and care should be taken to avoid breathing vapors. The mixture is heated to 104°C (220°F) at which point the solid phthalic anhydride is added. This will drop the temperature and the mixture is reheated to 104°C while stirring to dissolve the hardener. The mixture is stirred for 20 min after dissolving the hardener at 104°C. The mixture is cooled to 97°C (206°F) and placed in an oven preheated to this temperature. Every 30 min it is remixed for 3-5 min taking care not to mix in air bubbles. The viscosity is examined during each 3-5 min, stirring. After 2 h it will begin to thicken. It is important to notice the thickening, and once thickening starts to pour the mixture into the preheated molds immediately.

Thickening can be estimated by drawing off small samples from the mixture, cooling to room temperature, and pulling the sample into a long string. If the string breaks and snaps back, the mixture is ready to pour. Another indication of viscosity is gained by watching the runoff from the mixing rod onto the surface of the mixture. If it takes shape on the surface even for 1 s, the mixture is ready to pour.

Model Casting, Curing, and Loading

As suggested above, new molds should be cured at 5-10°C above the epoxy curing temperature. After 5 or 6 h at this temperature the oven temperature can be reduced to 93°C (200°F). When the molds reach this temperature, they are ready to receive the photoelastic material.

The mixture is poured, held for 2 h at 93°C, heated at 8°C/h to 116°C (240°F), held for 4 h, and then cooled at 8°C/h to 93°C (200°F). Further cooling below 93°C can be natural. This completes the preliminary curing of the material.

The models are taken from the oven and removed from the molds. The photoelastic material at this stage is a dark brown-green color. Some photoelastic birefringence may be present due to thermal stresses, which will be subsequently relieved in the stress freezing or final curing cycle. A unique property of the proposed material is its lack of brittleness at this stage. The material can be easily machined. The model can be prepared for loading and any needed machining performed. In any case it can be mounted in the loading fixture, thus, combining the final curing and the load cycle.

The model in the loading fixture is returned to the oven, heated to 124°C (255°F) at 11°C/h, held at 124°C for 4-6 h, and heated to the critical temperature of 135-140°C (275-285°F) at 8°C/h. The model is then loaded and cooled to 93°C (200°F) at a rate of 4°C/h or less. The oven can then be returned to room temperature at 7-11°C/h.

These heating and cooling rates were determined by trial and error to be the quickest which would not produce undesirable material properties, rind, or thermal stress. The cooling rate from 140 to 93°C, with the body loaded, is the slowest and most critical, since in this stage the photoelastic pattern due to load is frozen in and a thermal gradient would produce thermal stresses that would alter the desired pattern.

The fully cured material is a light yellow-brown. At the critical temperature (135-140°C) the modulus of elasticity is 18-19.4 MPa (2600-2800 psi), and the tensile strength is 0.7

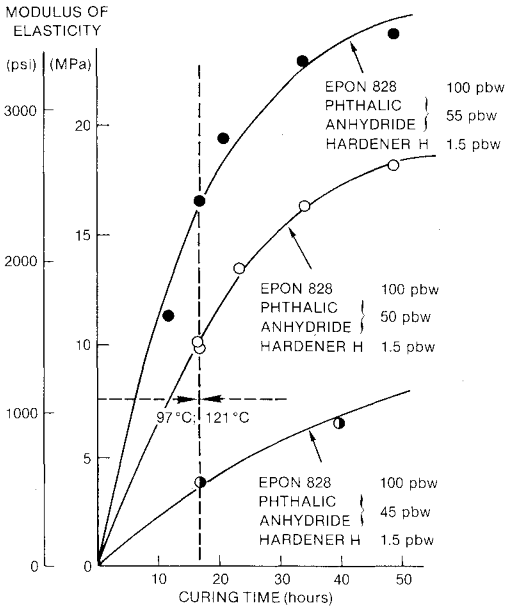

Fig. 2 Elastic modulus at critical temperature as function of curing time and resin/hardener ratio.

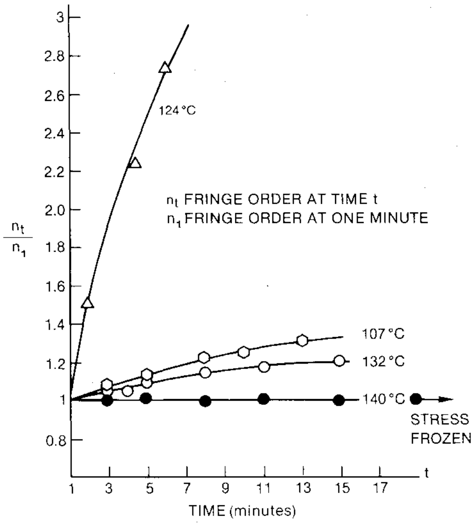

Fig. 3 Variation in fringe response with time for given stress at different temperatures.

MPa (104 psi). Figure 2 shows the modulus of elasticity for various resin/hardener ratios and times of curing. The material fringe constant is 0.32-0.38 KPa-m/fringe (1.8-2.2 psi-in./fringe). As indicated in Fig. 3 no optical creep is observed above 138°C (280°F). The pronounced optical creep in the temperature range 107-132°C indicates the transition of the material from the glassy state (below 107°C) to the rubber-like state about 132°C. The material is somewhat more brittle after the final curing than after preliminary curing, but still easily machined and sliced.

Because of the accelerated curing time and excellent machinability, the photoelastic material is recommended even if the problem requires a machined model (e.g., if no prototype exists). In such a case the model should be machined after preliminary curing. Elaborate precautions for machining three-dimensional photoelastic models such as those recommended in Ref. 5 are not deemed necessary. Models can be easily machined in production fixtures using ordinary production tools. For example, a hole drilled in the material without coolant using a common 1/2 in. shop drill, after preliminary curing (or even after final curing), will produce less than a fringe per 50 mm of material. Even this slight response can be annealed out. This is in marked contrast to the multiple rings of nonannealable fringes usually seen around drilled holes in other photoelastic materials.

Analysis

The model can be sliced, and the slices viewed and photographed in the polariscope in the usual way. No mottle has been observed in more than 30 different castings. Edge, or rind, effect is confined to about 0.5 mm from the surface and usually less than 0.06 fringes/1 mm of thickness.

Running Time

Many of the molds were made in one working day, and cured in a programmed oven overnight. The photoelastic castings were mixed and poured the second day and the preliminary curing completed in the same oven the second night. Models were prepared for loading and set in the loading fixture the third day. The combined curing-load cycle ran through the third night and into the fourth day. Slicing, viewing, and photographing and analysis were done on the fourth and fifth days to obtain stresses within one working week of the beginning of mold making. For additional castings of the same model the same molds were used a number of times. Thus, subsequent runs took one day less. More than 50 different geometries and loads have been analyzed using the new formulation over the last two years. Thirty of these jobs have been completed in 10 days or less.

The Rind Effect

The reduction in time over previous methods is in part due to the much shorter curing and loading cycles, but also due to the speeding up or elimination of machining. And this depends on elimination of rind. A quote from two decades ago[6] summarizes the problem:

"If all these models are to be produced by machining, as it was done in all the above quoted instances in order to eliminate a certain 'rind-effect' seemingly unavoidable in cast surfaces, the required operations may be extremely elaborate. For instance, the Rolls-Royce Company of England, in a remarkable series of tests on aircraft gas turbine components, found that the preparation of the photoelastic model of a centrifugal compressor rotor required 3-1/2 months of machining time on a fully programmed universal milling machine. Facilities of this nature are, of course, unavailable to most photoelastic laboratories. The simplest solution for overcoming difficulties of this type would be to prepare the model entirely by casting, since a prototype needed for this operation is usually provided in most cases. Thus far the above mentioned rind effect in the cast surfaces seems to have made this impossible. We find, however, one exception in the literature in a study by Kufner on the stress distribution in hydraulic turbine components. In this case, the models were cast directly into plaster molds with electrolytically deposited copper linings, and tested without additional surface machining."

Hetenyi[6] goes on to describe the study, noting striations in the material and concludes with the sentence, "Any development in this direction would enhance the use of photoelasticity in an unprecedented degree."

An extensive search was conducted to find a photoelastic material which would have negligible rind. In addition to the formulation finally chosen, two other materials considered for casting were Leven's material[7] and Sampson's material.[8]

Leven's formula is known to produce rind, and the rind is normally cut away before use. A study was made of casting Leven's formula with various protective atmospheres during curing. Castings during final curing were bathed in nitrogen, in silicone oil, and double wrapped in aluminum foil. All samples showed the same fringes due to rind as unprotected samples. The material that was cured in nitrogen was distinct from all the other samples in that the surface did not show the usual darkening associated with rind; however, the critical permanent fringes were still formed. Leven's material was also bathed in nitrogen during the preliminary curing and in both preliminary and final curing. Nitrogen in the preliminary curing had no influence on either surface color or fringes. This study demonstrated that surface oxidation at elevated temperatures is a distinct phenomenon not connected with rind effect.

Sampson's formula was cast and showed much less rind than Leven's formula. However, work was discontinued on this material because of the carciogenic nature of the hardener. The rind effect in Sampson's material was more than in the material finally chosen.

It is not yet completely clear why the proposed formulation gives so little rind effect. The addition of Hardener H, which is a mixture of aromatic amines, greatly decreases the curing time. It is felt that this accelerated curing does not allow time for the process responsible for the "rind effect" to occur. There are many hypotheses as to the cause of rind. They include molecular, chemical, mechanical, and thermal explanations as well as combinations of these. No useful purpose is seen in reviewing the possibilities.

For whatever reasons, the formulation prepared as explained above seems to have minimized the rind to an acceptable level. What rind could be detected was within about 0.5 mm from the cast surface and less than 0.06 fringe/1 mm of model thickness.

Replication

The proposed material gives excellent geometric replication of the prototype. One millimeter stampings and scratches on the surface are clearly duplicated in the model. Models of wooden prototypes show all the details of the wood grain. In models with fitted pieces, the model pieces fit each other and the prototype pieces with ease. One obvious exception is found across mold parting lines if the parts of the mold are not precisely rejoined after removing the prototype.

Difference in size between model and prototype depends not only on shrinkage of the curing epoxy, but also shrinkage of the mold material in the mold making, bulging of the mold during casting of the model, and, of course, ordinary thermal expansion and contraction. Comparison of model and prototype distances in regions without parting lines gave model dimensions 0.1-0.3% greater than the corresponding prototype dimensions, using all-silicone rubber molds.

Thick Models

When considering thermal response in casting plastics, as in casting metals, the most critical dimension is not the total volume nor some overall dimension, but rather the maximum wall thickness of the casting. Thus a diesel engine block (or its full-scale model) which has a maximum wall thickness of 2.5 cm (1 in.) is not an especially difficult casting problem. Neither is a 40 cm diameter model of a nuclear reactor, if the inside diameter is 30 cm or more. Volume does play a secondary role. Thus, a slab of a given thickness is more of a problem than a rod of a diameter the same as the thickness, and the rod will be more of a problem than a sphere of the same diameter. Still, for purposes of discussion, a thick model will be defined somewhat arbitrarily as one which has an interior point farther than 3 cm from the model surface.

In thick models, raising and lowering the temperature during curing and loading sets up thermal gradients, so that all parts of the model do not receive the same thermal cycle. In casting this may lead to partial curing and subsequent variation of optical and physical properties through the model (heterogeneity and anisotropy). Indeed, thick models can develop temperature change simply due to exothermic chemical reactions. Models 20-40 cm in diameter have been known to burn and even explode internally during curing.

During the load cycle the thermal gradients can lead to unwanted thermal stresses that can be frozen in with the desired stress field. The thicker the model the slower the temperature changes must be to minimize these effects.

For a particular material/model/mold combination the thickness should be considered. The recommended cycle was completely satisfactory for all the industrial models analyzed over the last two years that ranged up to 15 kg, 1.5 m in length, and 9.5 cm in thickness. Several models were specially cast to study thickness effects, using the recommended formula and cycles.

A cylinder 12 cm long by 12 cm in diameter was cast and cycled without load. No mottle, burning, rind, or other unwanted effects were observed in a center slice.

A $10 \times 13 \times 20$ cm block was cast and loaded on the small ends with a concentrated load. The curing and loading cycles were as specified above. In terms of the 10 cm thickness, these were very fast conditions and were meant as an extreme test of the method. The central slice taken from the block is shown in Fig. 4. Three features of the slice are noteworthy: mottle, edge effects, and uniform curing.

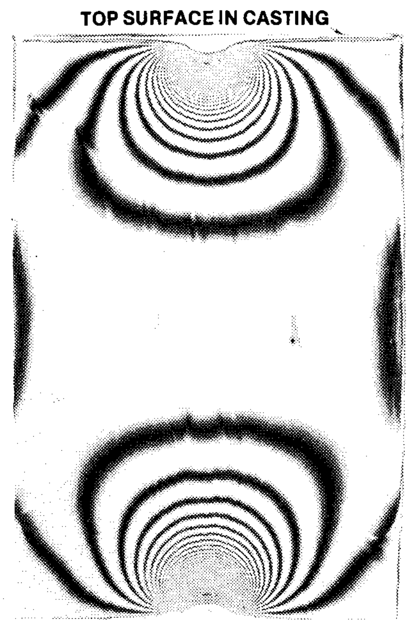

Fig. 4 Light-field isochromatics of central slice ($10 \times 20 \times 0.4$ cm thick) of $10 \times 13 \times 20$ cm block loaded on small ends.

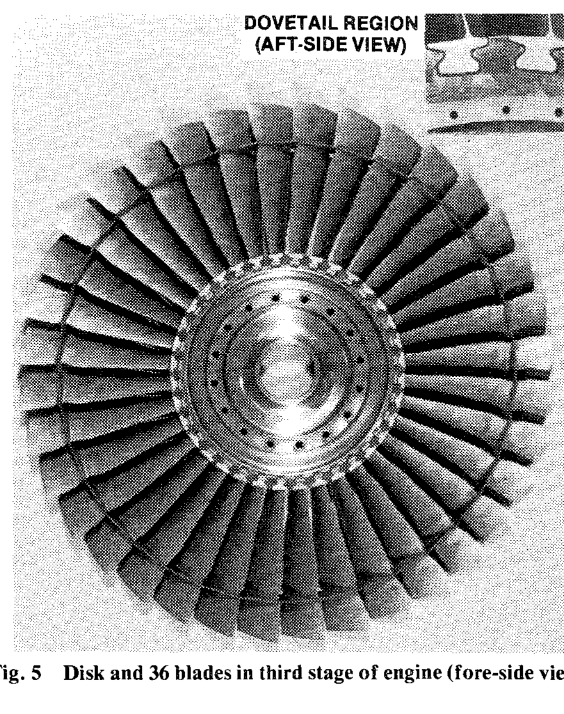

Fig. 5 Disk and 36 blades in third stage of engine (fore-side view).

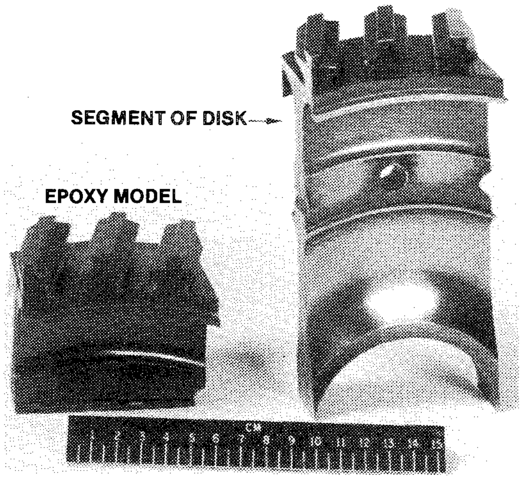

Fig. 6 Epoxy model of disk alongside disk segment from which it was replicated.

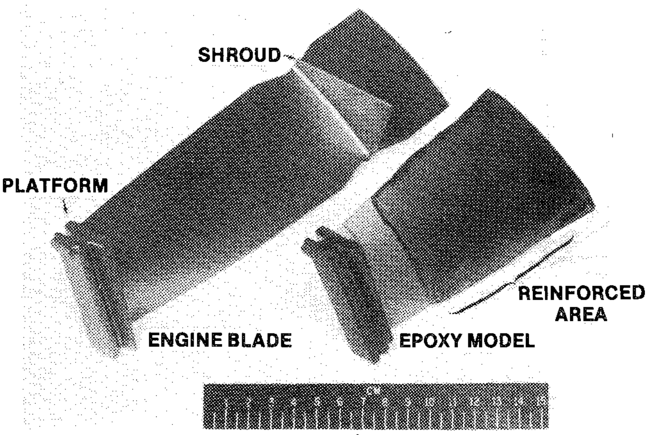

Fig. 7 Engine blade and replicated epoxy model.

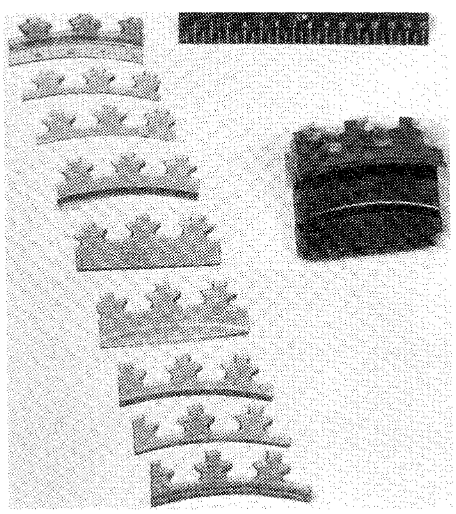

Fig. 8 Epoxy model and set of slices of the disk.

1) Mottle. The center of the slice exhibits a slight mottle pattern. This is probably due to the size of the batch and the size of the block, and is considered acceptable.

2) Edge Effect. The top surface shows more edge effect than the other three sides. This is expected since the mold was open at the top, and presumably phthalic anhydride evaporated from the top surface prior to and during curing. The edge effect on the other three sides may be attributed to rind, moisture absorption, or the small stresses induced in the

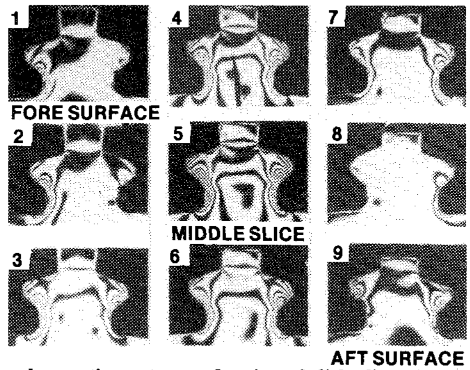

Fig. 9 Isochromatic patterns of series of disk slices (note variation from end to end).

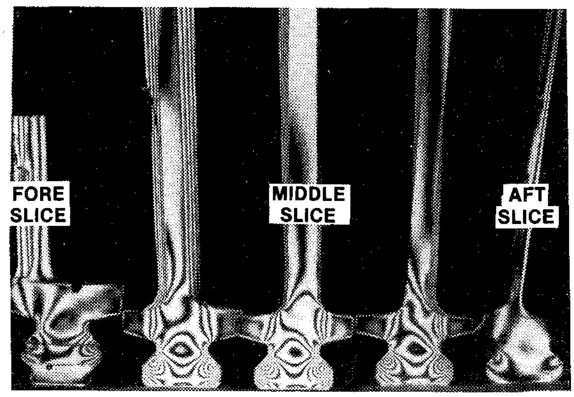

Fig. 10 Isochromatic patterns of series of blade slices.

rapid cooling. The fact that what little edge effect is seen is not due solely to rind was confirmed by annealing the slice. Annealing reduced the edge effect but rind cannot be annealed out.

3) Uniform Curing. With a large thickness, the region of questionable cure is the center. The color of the center is the same as near the surface. Within the approximations of the analysis, the fringe at the center of the block agreed with the fringe predicted using a 6 mm thick calibration disk from the same batch and the known applied stress.

The overall symmetry, continuity, and regularity of the pattern attests to the suitability of the method even for thick models. It may be that for this size and thickness the procedure should be modified to improve the analysis, but the tests as conducted gave acceptable results.

Turbine Blade Analysis

Figure 5 is an overview of the third-stage fan of a turbojet engine complete with 36 blades. The inset above shows the dovetail joint region treated in the three-dimensional photoelastic analysis. An extensive report of the analysis is given elsewhere.[9] Here an attempt will be made to demonstrate the advantages of the new photoelastic methods used in the study.

Within the region cited, the center of interest was the fillet of the disk lug teeth where failures had occurred. It was decided to model three disk lugs and two associated blades. A section containing three lugs was cut from the prototype disk. This section, along with two blades, were replicated in epoxy a number of times as described above. The blades were reinforced in the airfoil area to insure adequate area for gripping. Figures 6 and 7 show the prototypes and models.

Centrifugal force accounts for the primary load on the dovetail joints. Bending due to gas loads on the airfoil, vibrations, and acceleration also contribute to the loading. To analyze for the various possible loads it was decided to load models in three ways: 1) a load representing centrifugal force

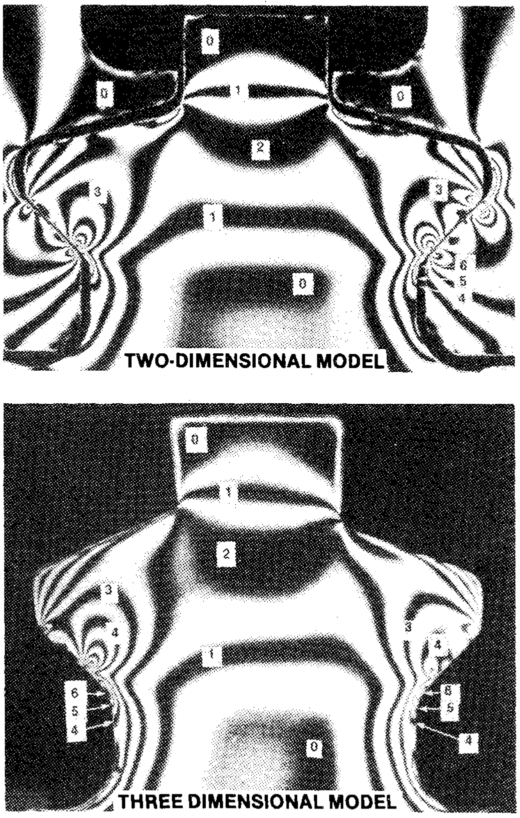

Fig. 11 Comparison of two- and-three-dimensional isochromatic patterns of disk lug.

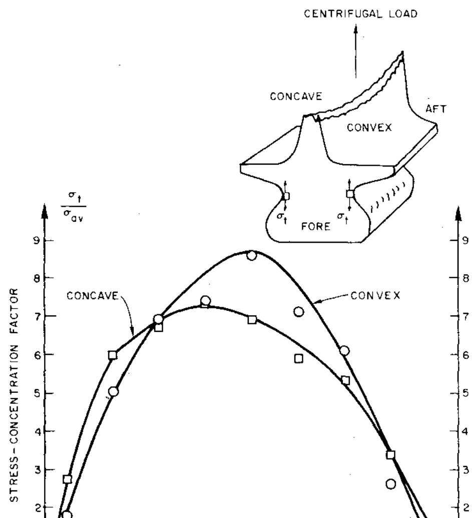

Fig. 12 Stress concentration factors in fillet of blade teeth under radial load.

alone; 2) a load representing a combination of centrifugal and bending in the plane shown in Fig. 5; and 3) a load representing a combination of centrifugal and bending load perpendicular to the plane shown in Fig. 5 (and through the disk axis). This required a minimum of three sets of models.

Loads were applied in a fixture that gripped the disk segment and applied simulated centrifugal and bending loads through thin cables set at various angles from the axes of the blade models and attached at the centroids of the blade models. To accommodate these loads three 6 mm mounting

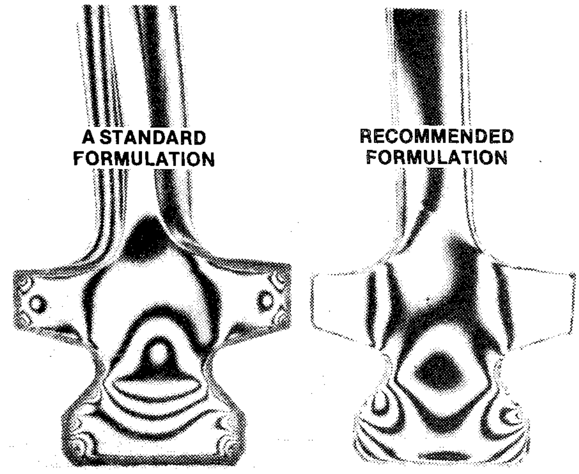

Fig. 13 Comparison of blade model slices of different epoxy formulations.

holes were drilled in each disk segment model and two 3 mm loading holes in each blade model about its centroid.

Seven batches of material were cast in the study. The first two batches were of simple shapes to study casting and loading techniques and edge effects. In each of the subsequent batches the prototype disk and blades were replicated. Three of the loadings were as described above. A fourth loading was an epoxy disk and prototype blades. The fifth loading was on a set of unreinforced blades.

All the models were sliced on a silicone-carbide parting wheel and received no further surface treatment. Figure 8 shows a series of slices taken from the disk model. Figures 9 and 10 are isochromatic patterns in slices cut from the disk and blade. Figure 11 compares a disk slice taken from the central part of the disk tooth with a two-dimensional model. The fringes in the three-dimensional pattern are not quite as sharp as their two-dimensional counterparts, but are fully analyzable and show no noticeable rind. The edges of the three-dimensional slice are not all perpendicular to the viewing plane. The slice was rotated to align both fillet edges as near as possible to the viewing plane. Thus, on the left edge of the model near the corner of the picture and on the upper surface of the tooth, the edge effects are due to inclination of the edge rather than rind.

Figure 12 shows the distribution of stresses in the fillets of the blades determined from the photoelastic patterns for the centrifugal loading. These and similar results were very useful in recommending redesign of the fillet areas. A finite-element study[10] of a similar problem is available for comparison.

Figure 13 is a comparison of two disk slices, one made from the proposed formulation and the other from a formulation used regularly in making machined models. Clearly the one pattern can be analyzed and the other cannot due to excessive rind effect.

Conclusion

A material and curing loading program is proposed for three-dimensional photoelastic analysis that should find application in industry where rapid analysis is essential. The method is found to give a high-quality, reliable stress analysis and is recommended for general use in both scientific and industrial research.

Acknowledgments

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